NUMERICAL MODELLING TO UNDERSTAND THE CAUSES OF THE EROSION AT BIG BAY, BLOUBERGSTRAND

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NON-TECHNICAL SUMMARY

CAUSES OF THE BEACH EROSION AT BIG BAY, BLOUBERGSTRAND

Introduction

Extensive erosion has occurred at Big Bay in Bloubergstrand, Cape Town over the last decade. During 2017 the City of Cape Town implemented a managed retreat of the dunes and walkways at Big Bay. However, ongoing erosion has led to further loss of beach amenity and is putting infrastructure such as the lifesaving building at severe risk, as seen below.

Eroded beach at Big Bay (April 2018).

It is unclear whether this erosion is cyclical and may reverse in future, or whether the Big Bay system has switched over to a long-term erosional state. PRDW Consulting Port and Coastal Engineers was appointed by the City of Cape Town to undertake a coastal processes study to address this question and inform decision making. The study was commissioned and funded by the City of Cape Town, while the preparation of a report to allow for wider dissemination of the study results was funded by the Big Bay Master Property Owners Association.
The primary objectives of the study were the following:

- Investigate the causes of the erosion at Big Bay, specifically whether it is due to a change in wave and/or wind climate which is found to cause either continuous erosion or alternating cycles of erosion/accretion; and
- Based on the results of the above analysis and the observed historical shoreline trends, predict the likely future shoreline erosion/accretion trends at Big Bay and Small Bay over the short (2019), medium (2025) and long term (2030), e.g. identify whether the existing erosional trend is most likely to continue, accelerate, stabilise or reverse.

**Methodology**

The methodology was to apply advanced computer models to simulate the waves, water levels, currents, sediment transport and erosion at Big Bay over the 39-year period from 1979 to 2018. Details are provided in the technical report.

**Results**

Erosion is the transport of sediment out of an area. When this occurs in Big Bay, the sea bed in the bay gets deeper and the beach gets smaller. To investigate the sediment transport dynamics of the Big Bay system, the average sediment transport rates were calculated over both summer and winter seasons over a period of 39 years. A snapshot of two sample years, one where the sediment movement is in balance and one where there was overall erosion, are presented in the figure below. The contours indicate the average sediment transport magnitude while the arrows indicate the direction that the sediment is moving.
Seasonal average sediment transport integrated over the winter and summer periods of a balanced year (2002/2003, top) and a recent erosional year (2016/2017, bottom).

In a balanced year (e.g. 2002/2003), the winter period (upper left plot) is characterised by a general southward sediment transport into Big Bay from the north. A northward transport out of the area occurs along the beach but is counteracted by the southward transport. The net result is an accretion of sediment in winter.

In the summer period (upper right plot), the transport rates are generally much lower and occur in shallower water due to smaller waves in summer. The northward transport along the beach remains similar to that in winter, but is not counteracted by a southward transport into this area, with the result that the beach experiences net erosion in summer. During this year the accretion in winter is balanced by the erosion in summer.
Considering the second example of an erosional year (2016/2017), the sediment transport patterns during the summer season (lower right plot) are similar to those for the balanced year, but the results in the winter season (lower left plot) reveal a significant reduction in the southward transport which would typically return sand to Big Bay. This imbalance results in a net loss of sediment and erosion in Big Bay. The sediment eroded from the beach is moved both offshore to the outer bay area as well as alongshore to the beaches north of Big Bay.

The numerical model results were used to calculate the changes in bed level over the 39.2 year period of available data. A time series of the result is shown below, where a positive change in bed level implies accretion and a negative change implies erosion.

*Modelled bed level change in Big Bay over the last 39 years. Trends are indicated by dotted lines in green (accretion), red (erosion), yellow (stabilisation) and grey (long-term trends).*

The bed level change in Big Bay suggests the presence of a long-term erosional trend over the last 20 years coupled with cyclical trends characterised by approximately four years of erosion followed by four years of accretion or stabilisation. These modelled trends agree with observations in the form of beach profile measurements and satellite imagery, providing confidence in the model results.

Analysis of the erosion/accretion cycles revealed a strong correlation with the offshore wave direction. During years when the offshore wave direction was more westerly, accretion occurred in Big Bay, while erosion occurred during periods during which the wave direction was more southerly. Small southerly shifts in wave direction were found to have a significant impact on the erosion.

Large waves in winter are generated by the frontal systems which approach the South African coastline from the south-west. Winter periods with more frequent and intense cold fronts are associated with more westerly waves and higher rainfall. A strong correlation was found between low rainfall years (using measured rainfall at Woodhead reservoir on Table Mountain) and more southerly waves which are associated with erosion of the beach. The opposite is true for wet years. Although correlation does not imply causality, the data suggests that the underlying causes of the recent drought in Cape Town and the erosion experienced at Big Bay are the same, i.e. a period during which winter storms were below average.
Conclusions

The results of the modelling indicated the following key conclusions in relation to the sediment transport dynamics in Big Bay:

- The sediment movement is driven by water currents and turbulence caused primarily by waves, with the wave direction determining the direction of the sediment movement.
- The inshore area of Big Bay typically loses sand northwards during summer due to more southerly waves in this season. Sand tends to return to Big Bay during winter when the waves are typically more westerly.
- Some winters have less westerly waves which leads to an annual imbalance, resulting in a net erosion.
- Historically, the duration of the erosion cycle has been approximately four years and is followed by a stabilisation or recovery of the beach.
- In addition to the four-year cycles, there has been a longer-term erosional trend over the last 20 years.
- The data suggests a correlation between erosion at Big Bay and drought in Cape Town, which is consistent with both being linked to years with a below-average occurrence of north-westerly winter storms.

In relation to the study objectives the following conclusions are drawn:

- The observed erosion/accretion trends in Big Bay are most likely caused by changes in wave direction, with cyclic erosion occurring due to a reduced occurrence of westerly waves during winters.
- The erosion over the last four years (2014 to 2017) is expected to begin to stabilise or reverse in the next one to two years, but it is unlikely to recover to levels seen prior to 1998.
1. INTRODUCTION

1.1 Background

Extensive erosion has occurred at Big Bay in Bloubergstrand, Cape Town over the last decade. During 2017 the City of Cape Town implemented a managed retreat of the dunes and walkways at Big Bay. However, ongoing erosion has led to further loss of beach amenity and is putting infrastructure such as the lifesaving building at severe risk, as seen in Figure 1-1.

![Figure 1-1: Eroded beach in front of the lifesaving building (April 2018).](image)

It is unclear whether this erosion is cyclical and may reverse in future, or whether the Big Bay system has switched over to a long-term erosional state. A coastal processes study is thus required to provide the City of Cape Town with a range of scenarios in respect of anticipated shoreline changes which can be used to inform decision making.

PRDW was appointed by the City of Cape Town to undertake the coastal processes study. The study was commissioned and funded by the City of Cape Town, with the results prepared as a presentation. The preparation of a report (this document) to allow for wider dissemination of the study results was funded by the Big Bay Master Property Owners Association. This technical report describes the methodology and findings of the study and includes a non-technical summary.
1.2 Study objectives

The primary objectives of the coastal processes study are the following:

- Investigate the causes of the erosion at Big Bay, specifically whether it is due to a change in wave and/or wind climate which is found to cause either monotonic erosion or cyclic erosion/accretion; and
- Based on the results of the above analysis and the observed historical shoreline trends, predict the likely future shoreline erosion/accretion trends at Big Bay and Small Bay over the short (2019), medium (2025) and long term (2030), e.g. identify whether the existing erosional trend is most likely to continue, accelerate, stabilise or reverse.

1.3 Modelling approach

Since two-dimensional (2D) sediment transport models are computationally expensive, long-term sand transport studies usually require the simplification of sediment transport processes. This is often achieved by employing a one-dimensional (1D) shoreline model (which assumes parallel depth contours and simulates longshore transport only) or by schematisation of the wave climate into an annual average climate described by a limited number of wave cases.

Big Bay is located in a complex wave environment – the regional wave climate is bi-modal, typically consisting of long-period south-westerly swell and either south-easterly or north-westerly locally generated wind waves. Moreover, Big Bay is located in the lee of Robben Island and therefore the wave climate at Big Bay is influenced by waves refracting around both the northern and southern sides of the island. The result is that even when the offshore wave climate is simple (e.g. swell waves only), the local wave climate will consist of waves coming from two directions. An example of this is shown in the satellite image of Big Bay presented in Figure 1-2.
In addition to the complex wave climate at Big Bay, the shoreline is not straight, but heavily curved and bounded by rocky outcrops, which means that 2D coastal processes are important and simplification to a 1D shoreline model is inappropriate. Furthermore, since the analysis of long-term trends is one of the primary objectives of this study, simplification of the wave climate into an annual average will defeat this purpose.

Considering the above, the following modelling approach has been developed for this study:

1. Model the wave transformation from deep water to obtain a long-term time series of wave conditions directly offshore of Big Bay.
2. Model the hydrodynamics and 2D sediment transport for a set of predetermined cases defined by the offshore wave conditions, the local water level and the longshore wind speed and direction.
3. Use a lookup function to match each time step in a long-term time series of waves, water levels and wind to the nearest modelled case.
4. Develop software to integrate the transport rates over time to determine 2D net transport rate and bed level changes for the duration of the long-term time series.
5. Develop software to aggregate the bed change to provide time series of bed change for a given area, such as Big Bay.

The advantage of this approach is that complex 2D sediment processes can be simulated in detail for individual cases while allowing the prediction of long-term sediment transport trends with the in-house developed software.

Since all cases are modelled using the same initial bathymetry, morphological feedback is not included in the model. Without morphological feedback the model cannot smooth out any irregularities in the input bathymetry. The effect of this is that the model results typically show large local trends where irregularities
in the bathymetry exist which would have been smoothed out if morphology could be included. Therefore, the model results should be used in an aggregated sense, e.g. bed change within an area rather than high-resolution spatial erosion/accretion plots. Furthermore, without morphological feedback, the model cannot directly investigate whether a tipping point has been reached since the bed remains fixed. Testing this would require the model to be run again using an eroded bathymetry. The advantage of not including morphological feedback is that the impact of long-term changes in the wave, wind and water level forcing can be isolated.

1.4 Report structure

Section 2 presents an analysis of historical cross-shore profile data collected in Big Bay. Section 3 provides the technical descriptions of the numerical models used in this study. The setup and results of the spectral wave modelling are presented in Section 4, while the setup and results of the coupled local 2D sediment transport modelling are presented in Section 5. Sections 6 and 7 present the conclusions and recommendations of this study. A list of references cited is given in Section 8.
2. ANALYSIS OF AVAILABLE BEACH PROFILE DATA

2.1 Big Bay cross-shore profiles: 2016-2018

Cross-shore beach profiles were measured by the City of Cape Town along the Big Bay shoreline between 14 March 2016 and 17 May 2018. Seventeen surveys were conducted over this period, including a survey shortly before and after the June 2017 storm. Figure 2-1 presents a plan view of the measured beach profile locations.

Figure 2-1: Measured beach profile locations.

The measured beach profiles were used to determine erosion and accretion trends over time along the Big Bay shoreline. Figure 2-2 presents the measured cross-shore profiles at Profile F, which is located in front of the lifesavers building. The lines on the figure are arranged from the oldest survey (blue) to the most recent survey (magenta). It is important to note that anthropogenic reshaping of the dune and cross-shore profile along Big Bay was undertaken between the October 2017 and February 2018 surveys. Changes to the profiles between these dates are therefore primarily due to the reshaping rather than natural cross-shore coastal processes.
Figure 2-2: Measured cross-shore profiles along Profile F.

The measured profiles show a clear gradual recession of the beach over the observed period. To quantify this trend, the horizontal movement of several vertical levels was tracked. Figure 2-3 and Figure 2-4 present the horizontal movement of the +0 m and +3 m MSL contours for each profile using the first survey as the baseline. Positive values indicate accretion and negative values indicate erosion. The lines on the figures are arranged from the southernmost profile (A, blue) to the northernmost profile (Q, magenta).
Figure 2-3: Time series of horizontal movement of the 0 m MSL contour.

Figure 2-4: Time series of horizontal movement of the +3 m MSL contour.
At both levels the profiles are generally observed to have eroded over the measurement period. However, as annotated on Figure 2-3, the shoreline change appears to occur over annual cycles with erosion occurring over the summer months and accretion during winter months.

The southernmost profiles do not show erosion at the +3 m MSL contour due to the presence of hard structures (e.g. gabions). The consequence is generally an increased erosion at the lower beach levels.

The horizontal movement shown in the figures above was calculated for the +0, +1, +2, +3 and +4 m MSL contours. The results were integrated to calculate a change in beach volume for each profile, expressed as m³/m. The results are presented in Figure 2-5.

Once again the results show an overall erosive trend across all profiles, while showing an accretive trend over the winter periods. While the June 2017 storm (shown as a black dotted line) is seen to have caused some erosion, the overall trend for the 2017 winter period remains accretive.

The time when the dune reshaping work was carried out is shown by the shaded grey rectangle. No change in the erosive trend is observed after completion of the works. This suggests that the erosion currently experienced is not a result of this intervention, but is rather a continuation of the ongoing erosional trend.

### 2.2 Table Bay beach profile data: 2005-2010

Beach profiles were also collected intermittently along the Table Bay coastline between 2005 and 2010, with approximately four surveys per year. One of the profile locations was inside Big Bay, approximately corresponding to Profile M shown in Figure 2-1.

In analogy to the analysis presented in Section 2.1, the shoreline change of this dataset was tracked over several vertical levels over the duration of the measured dataset. The analysis is presented in Figure 2-6,
which shows the horizontal excursion from a fixed beacon on land against time. Larger values indicate accretion, while smaller values indicate erosion.

![Graph showing shoreline movements](image)

**Figure 2-6**: Time series of horizontal movement of +0, +1, +2 and +3 m MSL contours near profile M over the period of 2005-2010.

The figure indicates a general accretive trend during this time. It is also worth noting that the profiles measured during or at the end of winter tend to be more accreted than those during summer, in agreement with the annual erosion/accretion cycles observed in the more recent profile data. This is especially pronounced in the movement of the +0 and +1 m MSL contours.

The shoreline movements described in this section are used to validate the model predictions – refer to Section 5.4.4.
3. MODEL DESCRIPTIONS

The numerical model used in this study is the *MIKE by DHI Flexible Mesh* model, which comprises a coupling between the following models:

- Spectral wave model;
- 2D Hydrodynamic model; and
- Sand transport model.

Descriptions of these models are given below.

3.1 Spectral wave model

The *MIKE 21 Spectral Waves (SW) Flexible Mesh* model was used for simulating wave transformation to the nearshore. The application of the model is described in the User Manual (DHI, 2017a), while full details of the physical processes being simulated and the numerical solution techniques are described in the Scientific Documentation (DHI, 2017b). The model simulates the growth, decay and transformation of wind-generated waves and swells in offshore and coastal areas using unstructured meshes.

In this study the model was run in the fully spectral formulation, including the following physical phenomena:

- Wind-wave generation;
- Dissipation due to whitecapping;
- Dissipation due to bottom friction;
- Dissipation due to depth-induced wave breaking;
- Refraction and shoaling due to depth variations; and
- The effect of time-varying water depth; the water depth at each time-step is computed in the hydrodynamic model.

The regional wave model was run in instationary mode, while the Table Bay and local coupled models were run in quasi-stationary mode.

3.2 Hydrodynamic model

The *MIKE 21 Flow Flexible Mesh* model was used for hydrodynamic modelling. The application of the model is described in the User Manual (DHI, 2017c), while full details of the physical processes being simulated and the numerical solution techniques are described in the Scientific Documentation (DHI, 2017d).

The model is based on the 2D shallow water equations, i.e. the depth-integrated incompressible Reynolds averaged Navier-Stokes equations. The time integration of the shallow water equations is performed using an explicit scheme. Horizontal eddy viscosity is modelled with the Smagorinsky formulation.
In this study, the model included the following physical phenomena:

- Currents due to tides;
- Currents due to waves: the second order stresses due to breaking of short period waves are included using the radiation stresses computed in the spectral wave model;
- Coriolis forcing;
- Bottom friction;
- Wind forcing; and
- Flooding and drying.

### 3.3 Sand transport model

The *MIKE by DHI Sand Transport* model was used for the sediment transport modelling. The application of the model is described in the User Manual (DHI, 2017e), while full details of the physical processes being simulated and the numerical solution techniques are described in the Scientific Documentation (DHI, 2017f).

The Sand Transport model calculates the transport of non-cohesive sediment (grain size > 0.063 mm) based on the combination of flow conditions from the hydrodynamic module and wave conditions from the spectral wave module. For the case of combined waves and currents, sediment transport rates are derived by linear interpolation in a sediment transport lookup table. The values in the table are calculated by the quasi three-dimensional sediment transport model (STPQ3D). The STPQ3D model calculates the instantaneous and time-averaged hydrodynamics and sediment transport in two horizontal directions. As the model calculates the bed load and the suspended load separately, the values in the sediment transport table are the total load.

The model accounts for graded sediment by dividing the grading curve into a number of size classes, calculating the sediment transport for each class and then averaging to obtain the total transport rate.

The temporal and vertical variations of shear stress, turbulence, flow velocity and sediment concentrations are resolved in the model. The time evolution of the boundary layer due to combined wave/current motion is solved by means of an integrated momentum approach. The force balance includes contributions from the near bed wave orbital motion, forces associated with wave breaking (gradients of radiation stresses) and the water level gradient.

The morphological development can be included by updating the bathymetry for every time step with the net sedimentation in each cell, which is based on the divergence of the sediment transport field and the porosity of the seabed. In order to reduce computer time, the morphological development can be sped up by multiplying the sedimentation by a speed-up factor. A varying sand layer thickness can be specified as the start condition for the simulation. This option is used when simulating sand transport in areas with rock bed present (i.e. cases with non-erodible bed and limited sand supply).
4. SPECTRAL WAVE MODELLING

4.1 Introduction

Wave transformation modelling was carried out to provide a long-term time series of wave conditions directly offshore of Robben Island. This time series was used to provide input to a detailed wave model of Table Bay, which in turn provided input wave conditions to a local coupled wave, hydrodynamic and sediment transport model of Big Bay. The setup and results of the regional wave model and Table Bay wave model are described in Section 4.2 and Section 4.3, while the coupled local 2D sediment transport model is discussed in Section 5.

4.2 Regional wave model

4.2.1 Mesh and bathymetry

The regional model mesh and bathymetry are shown in Figure 4-1 and Figure 4-2. The mesh extends approximately 50 km offshore of the Cape Peninsula to a depth of approximately -2 000 m Chart Datum (CD). The computational mesh comprised triangles with a resolution varying between approximately 8 km on the offshore boundary to approximately 2 km in Table Bay.

The bathymetry data was sourced from hydrographic charts available on the ‘MIKE by DHI CMAP Electronic Charts Database’ (DHI, 2017g) and from available nearshore bathymetric surveys surrounding the study area.

Figure 4-1: Regional model mesh.
4.2.2 Boundary conditions

Offshore hindcast wave data were obtained from the National Centers for Environmental Prediction (NCEP, 2012). Data from both the Reanalysis (1979-2009) and Operational (2005-2018) datasets were used to obtain a combined dataset of 39.2 years (1979-2018). The data comprises wave partition data on a 1 degree geographical grid at hourly intervals. The wave partition data characterises the sea state by identifying wave parameters ($H_m^0$, $T_p$, mean wave direction and directional spreading) for each of a variable number of peaks in the 2D wave spectrum. The wave partition data has been used to reconstitute the full 2D spectrum at each node and time step. The reconstituted spectra were applied as time and space-varying boundary conditions in the model.

Additional wind-wave generation was included in the model by applying a time and space-varying hindcast wind field over the model domain. The wind data is available from the National Centers for Environmental Prediction (NCEP, 2012) and comprises uninterrupted three-hourly wind speed and direction on a 0.5 degree geographical grid.

Bottom friction was modelled using a space-varying Nikuradse roughness varying between $k_N = 0.03$ m and $k_N = 1$ m to account for the rough seabed conditions characteristic along sections of the Cape Town Atlantic coastline.

The measured water level at the Port of Cape Town was applied as a time-varying, space constant water level. Gaps in the measured dataset were filled using the astronomical tide predicted from constituents determined from the measurements.
4.2.3 Results

The model was run for the full 39.2 year period over which the offshore boundary data was available. Data was extracted along the -100 m CD depth contour offshore of Robben Island. The extraction location is shown in Figure 4-2. A three-dimensional scatter plot of the 39.2 years of extracted wave parameters is presented in Figure 4-3.

![Three-dimensional scatter plot](image)

Figure 4-3: Three-dimensional scatter plot of significant wave height ($H_{m0}$), Peak wave period ($T_p$) and Mean Wave Direction (MWD) of 39.2 years of data extracted at a depth of -100 m CD offshore of Robben Island.

The scatter plots were used to identify a set of cases adequately describing the wave climate of interest to this study. Care was taken to provide sufficient resolution of the larger and more oblique wave events, since these events are typically dominant in the sediment transport regime. 44 cases were selected and are shown as yellow dots on the plots.
4.3 Table Bay wave model

4.3.1 Mesh and bathymetry

The mesh and bathymetry used in the Table Bay wave model are shown in Figure 4-4 and Figure 4-5. The mesh extends offshore to the -100 m CD depth contour where the wave boundary conditions were extracted from the regional wave model. The mesh comprised triangular elements with a resolution varying from 300 m offshore to approximately 100 m in areas of interest or rapidly varying bathymetry such as at Whale Rock, south of Robben Island.

Figure 4-4: Table Bay model mesh.
4.3.2 Boundary conditions

The selected wave cases determined from the results of the regional wave model (Figure 4-3) were applied along the offshore boundary of the Table Bay model. Each of the 44 cases was modelled as a discrete event at four different constant water levels across the tidal range, resulting in a total of 176 cases. The selection of the water levels is discussed in Section 5.2.1.

Additional wind generation within Table Bay was not included in the model. Bottom friction was modelled using the same space-varying Nikuradse roughness used for the regional wave model discussed in Section 4.2.2.

4.3.3 Results

The wave transformation for a south-westerly storm is shown in Figure 4-6.
Figure 4-6: Wave transformation into Table Bay for a south-westerly storm ($H_m = 5.5 \, m$, $T_p = 12.5 \, s$, $MWD = 210^\circ$, Water level = +1.0 m CD). Contours indicate significant wave height, while vectors indicate the mean wave direction.

Clearly visible on the figure is the sheltering effect to the north-east of Robben Island. In the lee of the island, the wave climate consists of energy coming around both the north and around the south of the island, similar to that observed in the satellite image presented in Figure 1-2. The vectors shown in the figure represent the mean wave direction and thus appear to be approximately shore-normal closer to Big Bay even though the fully spectral model properly accounts for wave energy from both directions.

An example of the wave transformation during a more westerly storm event is shown in Figure 4-7.
Figure 4-7: Wave transformation into Table Bay for a westerly storm ($H_m = 5.5$ m, $T_p = 12.5$ s, $MWD = 255^\circ$, Water level = +1.0 m CD). Contours indicate significant wave height, while vectors indicate the mean wave direction.

Due to the higher bed roughness south of Robben Island compared to the north, more energy is seen to reach Big Bay under the more westerly wave conditions.

The Table Bay wave model was run for all 176 combinations of wave and water levels to simulate the transformation to Big Bay. For each combination the fully-spectral wave conditions offshore of Big Bay were extracted from the model results for use in the local coupled model discussed in Section 5.
5. 2D SEDIMENT TRANSPORT MODELLING

5.1 Introduction

Recall from Step 2 of the modelling approach presented in Section 1.3 that the 2D hydrodynamics and sediment transport are modelled for a set of predefined cases. The results of these cases are then integrated over the long term to provide 2D net sediment transport rates and cumulative bed level changes over time.

The selection of the cases to be modelled is described in Section 5.2. The setup of the coupled wave, hydrodynamics and sediment transport model is presented in Section 5.3, while the results are presented in Section 5.4 and discussed in Section 5.5.

5.2 Selection of cases

5.2.1 Water levels

Water levels measured at the Port of Cape Town were available for the period of 1967-2018. The measured water levels are dominated by tidal variations, but also include tidal residuals such as storm surge. This measured dataset was used to characterise the water levels offshore of Big Bay.

A tidal harmonic analysis was carried out on the measured data, using the Tidal Analysis of Heights tool in the MIKE 21 Toolbox (DHI, 2017h). The output of the tool is a set of tidal constituents for the site. Using the same tool, the tide at the Port of Cape Town was predicted using the constituents obtained from the analysis. Gaps in the measured data were filled using the predicted tide. An exceedance curve of the filled data is shown in Figure 5-1.

![Figure 5-1: Exceedance curve of measured water levels at the Port of Cape Town.](image-url)
Four water level cases of +0.5, +1, +1.5 and +2 m CD were selected, shown as yellow dots on the figure. While the first three cases form a symmetrical representation of the water levels, the fourth and highest case was added to ensure adequate representation of high water levels which often occur with large storm events.

5.2.2 Waves

The selection of the wave cases has been discussed in Section 4.2.3.

5.2.3 Wind

The sediment transport dynamics at Big Bay are influenced not only by waves and water levels, but also by wind-driven longshore currents. To include this effect, wind forcing was included in the hydrodynamic model.

Hindcast wind data was extracted from the NCEP global hindcast database at the node nearest to Big Bay (34°S 18°E). The dataset comprises uninterrupted three-hourly wind speed and direction for the 39.2 year period of 1979-2018. Based on the shoreline orientation at Big Bay, the wind speed and direction were converted to a longshore component, with positive values indicating wind blowing from south to north along the shoreline. An exceedance curve of the longshore component of the wind speed is shown in Figure 5-2.

![Figure 5-2: Exceedance curve of longshore component of wind speed. Positive values represent winds blowing from south to north along the shoreline.](image)

The strongest longshore wind components with speeds up to 25 m/s are negative (i.e. blowing from north to south), and are typically associated with winter storms. However, negative longshore winds occur for only approximately one third of the time. For the remainder of the time the wind blows from south to north, typically associated with south-easterly wind conditions. These winds are somewhat weaker with a maximum longshore component of approximately 22 m/s.

Seven wind cases were selected to resolve the range of possible wind conditions. These are shown as the red dotted lines in Figure 5-2, with three negative cases, three positive cases and one calm case (i.e. no wind).
5.2.4 Selection of modelled combinations

The 44 wave cases, 4 water level cases, and 7 wind cases result in a total 1232 possible combinations. However, after removing combinations which have not occurred in the 39.2 year time series, 839 cases remain to be modelled.

5.3 Model setup

5.3.1 Mesh and bathymetry

The Big Bay coupled model mesh and bathymetry are shown in Figure 5-3 and Figure 5-4.

The mesh extends approximately 2.3 km north and south of Big Bay and approximately 1.5 km offshore of Big Bay to a depth varying between -15 m CD and -12 m CD along the offshore boundary. The mesh comprises triangular and quadrangular mesh elements and has a resolution varying between 50 m at the offshore boundary to 7.5 m along the shoreline in Big Bay.

The data sources used to define the bathymetry included the following:

- Lidar data (2009);
- Beach profiles (2016);
- Several surveys of Table Bay, including some data in Big Bay and Small Bay (2006); and
- C-MAP Electronic Charts (DHI, 2017g).

![Image of mesh and bathymetry](image_url)

Figure 5-3: Big Bay coupled model mesh.
5.3.2 Waves

Fully spectral, space-varying boundary conditions were extracted from the Table Bay spectral wave model for each of the wave and water level combinations, and were applied along the three model boundaries.

Bottom friction was modelled using a constant Nikuradse roughness of $k_N = 0.1$ m. Water levels for the wave model were obtained from a direct coupling with the hydrodynamic model.

The spectral wave model was run in the fully spectral, quasi-stationary formulation, excluding additional wave generation due to wind forcing.

5.3.3 Hydrodynamics

Constant water levels were applied along the boundary of the hydrodynamic model, corresponding to the water level of the relevant case. Wind-generated currents were included by specifying the longshore wind component depending on the case modelled and using a wind friction coefficient linearly varying with wind speed.

Wave-driven currents were included through a direct coupling with the spectral wave model. Bed resistance was modelled using a Chezy formulation, with a space-varying Chezy number of $50 \text{ m}^{1/2}/\text{s}$ in sandy areas and $20 \text{ m}^{1/2}/\text{s}$ where rough exposed rocks and rocky reefs are present.

5.3.4 Sediment transport

A median sediment grain diameter of $d_{50} = 0.21$ mm with a grading coefficient ($\sqrt{d_{84}/d_{16}}$) of 1.3 was used, based on an analysis of sediment samples from Big Bay.

A space-varying sediment thickness was specified over the model domain to account for reduced sediment transport over areas with a rocky seabed. These areas include the rocky outcrops and other exposed reefs identified on satellite images with good visibility of the seabed. Morphological feedback (i.e. dynamic updating of the bed level) was not included, since the model was used to determine the steady state sediment transport rates for each case.
The STPQ3D model (refer to Section 3.3) was used to generate the transport tables used in the sand transport model. The STPQ3D model input parameters were specified as follows:

- Critical Shields’ parameter = 0.05;
- Bed ripples were excluded;
- Stokes’ 1st order wave theory was used;
- Deterministic bed concentration formulation was used; and
- Wave breaker index = 0.8.

Figure 5-5, Figure 5-6 and Figure 5-7 show examples of the transport rates for different current speeds and wave heights at depths of 1 m, 2 m and 5 m, respectively. In each case, the curve for the largest wave height corresponds to the depth-limited wave, i.e. 0.8 times the water depth. Note that the sediment transport rate is plotted on a logarithmic axis. The curves clearly indicate the non-linear increase in the sediment transport rate with a linear increase in current speed.

![Graph showing sediment transport rate vs. current speed for different wave heights at 1 m depth.](TestTransportTable01.xlsx, ad_Scaled3.0 Detailed, Depth=1m)

**Figure 5-5**: Modelled sediment transport rate due to currents and waves in a water depth of 1 m. 

\[ d_{50} = 0.21 \text{ mm}. \]
Figure 5-6: Modelled sediment transport rate due to currents and waves in a water depth of 2 m. 
\(d_{50} = 0.21\) mm.

Figure 5-7: Modelled sediment transport rate due to currents and waves in a water depth of 5 m. 
\(d_{50} = 0.21\) mm.
The model was run to produce a steady-state sediment transport solution for each of the 839 modelled cases.

5.4 Results

5.4.1 Individual results

Figure 5-8, Figure 5-9 and Figure 5-10 present the steady state wave transformation, currents and sediment transport during a typical south westerly wave event at a water level of +1 m CD and with no longshore wind. The sediment transport contours and vectors show the sediment transport magnitude and direction on a logarithmic scale.

Figure 5-8: Wave transformation during a south-westerly wave event (offshore $H_m = 5.5$ m, $T_p = 12.5$ s, Direction = 210°, Water level = +1.0 m CD, No wind).
Figure 5-9: Steady-state currents during a south-westerly wave event (offshore $H_{m0} = 5.5$ m, $T_p = 12.5$ s, Direction = 210°, Water level = +1.0 m CD, No wind).

Figure 5-10: Steady-state sediment transport during a south-westerly wave event (offshore $H_{m0} = 5.5$ m, $T_p = 12.5$ s, Direction = 210°, Water level = +1.0 m CD, No wind).
The wave transformation shows wave energy propagating into Big Bay. Wave breaking is seen to occur over the rocky outcrops north and south of Big Bay, causing strong wave driven currents in these areas as seen in the current speed plot. The currents also exhibit a complex 2D pattern in Big Bay including eddies and rip currents.

The sediment transport plot shows a general northward sediment transport pattern along the studied coastline. The complex 2D currents observed in Figure 5-9 result in similarly complex 2D sediment transport patterns. However, the general northward transport trend remains.

The same outputs are shown in Figure 5-11 to Figure 5-13 for a more westerly wave event at the same water level and again with no wind.

*Figure 5-11: Wave transformation during a westerly wave event (offshore $H_{\text{m0}} = 5.5$ m, $T_p = 12.5$ s, Direction = 255°, Water level = +1.0 m CD, No wind).*
Figure 5-12: Steady-state currents during a westerly wave event (offshore $H_{\text{m0}} = 5.5$ m, $T_p = 12.5$ s, Direction = $255^\circ$, Water level = $+1.0$ m CD, No wind).

Figure 5-13: Steady-state sediment transport during a westerly wave event (offshore $H_{\text{m0}} = 5.5$ m, $T_p = 12.5$ s, Direction = $255^\circ$, Water level = $+1.0$ m CD, No wind).
The wave directions seen on the wave transformation plot show an oblique angle of approach which can be expected to drive a southward longshore current. This is confirmed in the current speed plot, which shows strong southward currents, especially north of Big Bay where the wave heights are the largest. There remain some complex current patterns in the bay, resulting in northward currents along the shoreline. The steady-state sediment transport patterns also show a general southward trend, with complex patterns in the bay.

5.4.2 Post-processing of model results

A long-term time series of input conditions was compiled from the following sources:

- Modelled wave data for the period of 1979-2018 (hourly) extracted offshore of Robben Island at a depth of -100 m CD;
- Water level data measured at the Port of Cape Town for the period of 1967-2018 (hourly); and
- Hindcast wind data from NCEP (2012) extracted for the period of 1979-2018 (three-hourly, interpolated to hourly intervals) at the node nearest to Big Bay.

The combined overlapping period is 39.2 years (1979-2018, hourly).

Two different software programs were developed in-house for the post-processing of the results. The first program was developed to calculate the integrated net sediment transport rate and resulting bed level change in each mesh cell over time. This was achieved by first matching each time step of the 39.2 years of input conditions to the nearest of the 839 modelled cases. Then the modelled transport rates and bed changes were integrated to produce the net transport and bed change at given intervals (e.g. annually, seasonally, monthly or weekly).

As explained in the modelling approach (Section 1.3), due to the exclusion of morphology in the sediment transport model the results should be used in an aggregated sense, e.g. bed change within an area rather than high-resolution spatial erosion/accretion plots. The second program was developed to calculate the average bed change over a given area. The output of the program is a time series of cumulative bed level change at regular intervals (e.g. monthly).

5.4.3 Seasonal net transport rates

To investigate the sediment transport dynamics of the Big Bay system, the seasonal net transport rates were integrated over a six-month winter and a six-month summer period of a balanced year using the developed in-house software. The winter periods were defined as May to October and the summer periods as November to April. The results are presented Figure 5-14. The contours indicate the net transport magnitude while the vectors indicate the net transport direction. Note that nonlinear scales are used. The magenta dotted lines indicate the areas (Big Bay Outer and Big Bay Inner) over which the bed change has been integrated for further assessment.
Figure 5-14: Seasonal net sediment transport integrated over the winter (left) and summer (right) periods of a balanced year (2002/2003).

The winter period is characterised by a general southward transport into Big Bay from the north. In the Big Bay Inner area, a northward transport out of the area occurs along the beach but is counteracted by the southward transport. The net result is an accretion of sediment in this area. The Big Bay Outer area is fed by sediment from Big Bay Inner, but the southward transport out of the area is dominant, resulting in a net loss of sediment.

In the summer period, the transport rates are generally much lower and occur in shallower water due to smaller waves in the summer season. In Big Bay Inner, the northward transport along the beach remains similar to that in winter, but is not counteracted by a southward transport into this area, with the result that this area experiences net erosion. Although not visible at the scale shown, the net transport into the Big Bay Outer area exceeds the transport out, resulting in accretion in this area.

To aid understanding of the recent erosion occurring at Big Bay, the same plots were produced for a recent year as presented in Figure 5-15.
Figure 5-15: Seasonal net sediment transport integrated over the winter and summer periods of a recent year (2016/2017).

While the transport patterns during the summer season are similar to those for the balanced year, the trends in the winter season reveal a significant reduction in the southward transport which would typically return sand to Big Bay. This imbalance results in a net loss of sediment in the Big Bay Inner area and associated beach erosion.

5.4.4 Long-term bed level change

Using the in-house software, the cumulative average bed level change in the Big Bay Inner and Outer areas was integrated at monthly intervals over the 39.2 year period of available data. A time series of the result is shown in Figure 5-16.
Figure 5-16: Time series of cumulative average bed level change over time for the Big Bay Inner and Big Bay Outer areas: full 39.2 year data (top) and 10-year detail (bottom). The areas are shown in Figure 5-14.

In the long term, Big Bay Inner shows a net erosion since approximately 1998, while a gradual accretion of Big Bay Outer is observed. The time series for Big Bay Inner indicates a period of accretion between 2005 and 2010 and erosion over the period of 2015 to 2018. These trends agree very well with those determined from the analysis of cross-shore profiles measured over the same periods (presented in Section 2), increasing confidence in the model results.

The seasonality in transport patterns observed in Section 5.4.3 is again highlighted in the one-decade detail. The Big Bay Inner area is seen to experience accretion during the winter season and erosion during the summer season. Conversely, Big Bay Outer experiences gradual accretion during summer and erosion during large winter storms. As for the long-term trends, the seasonal cycles are also supported by observations from the analysis of measured profiles, which indicated periods of accretion during winter and erosion during summer along the upper beach profiles which form part of Big Bay Inner.

Satellite images were analysed in an attempt to ground truth the observed long-term trends over the last two decades. Figure 5-17 presents a comparison of an image from 2005 with one from 2017. The extent of sediment cover over rocky outcrops was used as an indicator of sediment levels in the bay. Red dotted circles indicate areas where erosion occurred while green ones indicate accretion.
Figure 5-17: Comparison of satellite imagery from 2005 and 2017 showing areas where erosion (red) and accretion (green) have occurred. Erosion/accretion patterns were derived from the extent of sediment cover over rocky outcrops.

Although the images present only a snapshot in time, the changes in sediment level support the observed trends of erosion in Big Bay Inner and accretion in Big Bay Outer (and Small Bay) over the period of 2005 to 2017.
5.5 Discussion

The long-term time series of bed level change presented in Figure 5-16 showed multi-year periods of erosion or accretion. In an effort to understand the underlying causes of these cycles, the bed level change in Big Bay Inner is plotted in Figure 5-18 together with the long-term time series of offshore (-100 m CD) annual average significant wave height, peak wave period and mean wave direction for each calendar year. A long-term trend line is also drawn through the wave direction.

![Figure 5-18: Comparison of Big Bay Inner average bed level change with offshore wave conditions.](image)

The best correlation is observed with the offshore wave direction. During periods when the offshore annual average wave direction was more westerly, accretion occurred in Big Bay, while the opposite is true for periods during which the annual average direction was more southerly. The overall trend in the annual average wave direction suggest a gradually southward rotating wave climate. Judging by the effect of more southerly waves on individual years, this trend could contribute to the gradual long-term erosion of Big Bay Inner seen in Figure 5-16.
Large waves in winter are generated by the frontal systems which approach the South African coastline from the south-west. Winter periods with more frequent and intense cold fronts are associated with more westerly waves and higher rainfall. Since cold fronts are also the primary source for rainfall in Cape Town, this was investigated by comparing the trends in the wave direction above with the measured rainfall at Woodhead reservoir on Table Mountain. The measured rainfall data was available from the Department of Water and Sanitation (DWS, 2019). Figure 5-19 presents a scatter plot of the modelled annual bed level change in Big Bay against annual rainfall.

![Figure 5-19: Scatter plot of modelled annual bed level change in Big Bay vs annual rainfall measured at the Woodhead reservoir on Table Mountain.](image)

The data shows erosion in ten of the thirteen years with low rainfall and accretion in ten of the thirteen years with high rainfall. Although correlation does not imply causality, the data suggests that the underlying causes of the recent drought in Cape Town and the erosion experienced at Big Bay are the same, i.e. a period during which winter storms were below average.

Further investigation of the long-term bed level change at Big Bay Inner (Figure 5-20) suggests the presence of a long-term erosional trend over the last 20 years coupled with cyclical trends characterised by approximately 4 years of erosion and 4 years of accretion or stabilisation.
Figure 5-20: Identification of trends in modelled long-term average bed level change of Big Bay Inner. Trends are indicated by dotted lines in green (accretion), red (erosion), yellow (stabilisation) and grey (long-term trends).
6. CONCLUSIONS

The results of the modelling indicated the following key conclusions in relation to the sediment transport dynamics in Big Bay:

- The inshore area of Big Bay typically loses sand northwards during summer due to more southerly waves in this season.
- Sand tends to return to Big Bay during winter when the waves are typically more westerly.
- Some winters have less westerly waves which leads to an annual imbalance, resulting in a net erosion.
- Historically, the duration of the erosion cycle has been approximately 4 years and is followed by a stabilisation or recovery of the beach.
- In addition to the 4 year cycles, there has been a longer-term erosional trend over the last 20 years.
- The offshore area of Big Bay has tended to accrete due to erosion of the inner area, with some of the sand lost further offshore during extreme storm events.
- The data suggests a correlation between erosion at Big Bay and drought in Cape Town, which is consistent with both being linked to years with a below-average occurrence of north-westerly winter storms.

In relation to the study objectives the following conclusions are drawn:

- The observed erosion/accretion trends in Big Bay is most likely caused by changes in wave direction, with cyclic erosion occurring due to a reduced occurrence of westerly waves during winters.
- Based on the results of the study, the erosion over the last four years (2014 to 2017) is expected to begin to stabilise or reverse in the next one to two years, but it is unlikely to recover to levels seen prior to 1998.
7. RECOMMENDATIONS

The following recommendations for future work are made:

- It is recommended that the model be used to test whether the erosion of the beach and foreshore has caused a tipping point where the offshore rocks no longer hold the beach salients and the erosion will then continue until a new equilibrium is reached, independent of the changes in the wave climate described in the conclusions. This would have serious implications for decision making and protection of infrastructure.

- The spectral wave model should be updated to include calibration against wave measurements in Table Bay.

- The Big Bay coupled model should be updated by including the initial morphological response in the modelled bathymetry, which will facilitate smoother and more spatially detailed results.
8. REFERENCES


